Functions from a Calculus Perspective

1-7: Inverse Relations and Functions

Common Core State Standards: A-SSE.2, F-BF.4b,c,d.

Objectives:

- Use the horizontal line test to determine inverse functions.
- Find inverse functions algebraically and graphically.

I. Inverse Functions: Functions whose domain and range are switched and the result f^1 is a function as well.

Inverse functions numerically:

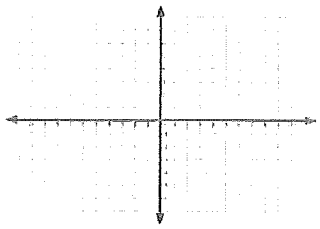
f(x)	$f^{-1}(x)$

Inverse functions graphically:

f(x)	$f^{-1}(x)$

Inverse functions algebraically:

f(x)	$f^{-1}(x)$



1.What is a function for the area of a square?

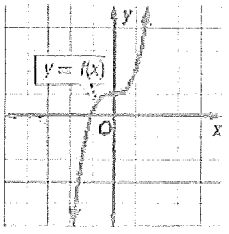
2. What is the area of a square when the side measures 5?

3. Write a function that represents the side of a square given the area.

What is the length of the side of a square if the area of the square is 100?

4. Write a function for distance if rate is constant and time is variable. Write a function for time if distance is variable and rate is constant.

5. Use the graph of relation A to sketch the graph of its inverse:



Will every function that passes the vertical line test have an inverse that is a function?

Take a look at the parabola $y = x^2$.

Does it pass the VLT? What does this indicate?

Now solve for the inverse numerically, geometrically & algebraically.

Numerically: Geometrically: Algebraically:

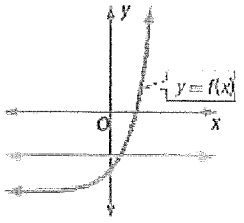
What can you conclude? Is this a function? Explain.

A new test! Horizontal Line Test

Key Concept Horizontal Line Test

Words	A function f has an inverse function f^{-1} if and only if each horizontal line intersects the graph of the function in at most one point.
Example	Since no horizontal line intersects the graph of f more than once, the inverse function f^{-1} exists.

Model



Tip! Remember, any function that passes both the vertical and horizontal line test is a one to one function!

1.

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

a. $y = 4x^2 + 4x + 1$

b. $f(x) = x^5 + x^3 - 1$

II. Finding an Inverse Function

KeyConcept Finding an Inverse Function

Step 1 Determine whether the function has an inverse by checking to see if it is one-to-one using the horizontal line test.

Step 2 In the equation for $f(x)$, replace $f(x)$ with y and then interchange x and y.

Step 3 Solve for y and then replace y with $f^{-1}(x)$ in the *new* equation.

Step 4 State any restrictions on the domain of f^{-1}. Then show that the domain of f is equal to the range of f^{-1} and the range of f is equal the domain of f^{-1}.

2.

Determine whether f has an inverse function. If it does, find the inverse function and state any restrictions on its domain.

a. $f(x) = \frac{x}{2x - 1}$

b. $f(x) = 2\sqrt{x - 1}$

III. Showing that functions are inverses: Double Composition.

KeyConcept Compositions of Inverse Functions

Two functions, f and g, are inverse functions if and only if

- $f[g(x)] = x$ for every x in the domain of $g(x)$ and
- $g[f(x)] = x$ for every x in the domain of $f(x)$.

3.

Show that $f(x) = \frac{2}{3}x + 2$ and $g(x) = \frac{3}{2}(x - 2)$ are inverse functions.

4.

MANUFACTURING The fixed costs for manufacturing one type of stereo system are $96,000 with variable cost of $80 per unit. The total cost $f(x)$ of making x stereos is given by $f(x) = 96{,}000 + 80x$.

a. Explain why the inverse function $f^{-1}(x)$ exists. Then find $f^{-1}(x)$.

b. What do $f^{-1}(x)$ and x represent in the inverse function?

c. What restrictions, if any, should be placed on the domain of $f(x)$ and $f^{-1}(x)$? Explain:

d. Find the number of stereos made if the total cost was $216,000.

5. Does the function f(x) = [x] have an inverse that is a function? Explain.

Section 1-7 HW: P. 70-72 #1-7 (odd), 13-23 (odd), 27-33 (odd), 37-39 (odd), 47-51