+ slope of tangent line

Chapter 1 Review Name: Key 2014-15 Hr______

Section 1-1 Content

For #1-4, write each set of numbers in set-builder and interval notation, if possible.

1. $\{-3, -2, -1, 0, 1, ...\}$ set builder

$\{x \mid x \geq -3, x \in \mathbb{Z}\}$

2. $-6.5 < x \leq 3$ Interval

$\{x \mid -6.5 < x \leq 3, x \in \mathbb{R}\}$; $(-6.5, 3]$

3. all multiples of 2 set builder

$\{x \mid x = 2n, n \in \mathbb{Z}\}$

4. $x < 0$ ~~and~~ or $x > 8$ Interval

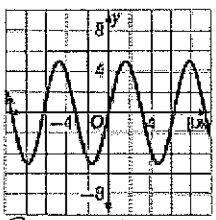
$\{x \mid x < 0 \text{ or } x > 8, x \in \mathbb{R}\}$; $(-\infty, 0) \cup (8, \infty)$

For #5-7, determine whether each relation represents a function of x.

5. The input value is a car's license plate number, and the output value y is the car's make and model.

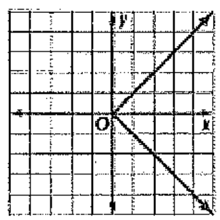
function

6.



function

7.



Not a fn'

8. $-x + y = 3x$

$y = 4x$

function

For #9-10, find each function value.

9. $h(x) = x^2 - 8x + 1$

a. $h(-1) = (-1)^2 - 8(-1) + 1$

$= 1 + 8 + 1$

$= 10$

b. $h(2x) = (2x)^2 - 8(2x) + 1$

$= 4x^2 - 16x + 1$

c. $h(x+8) = (x+8)^2 - 8(x+8) + 1$

$= (x^2 + 16x + 64) - 8x - 64 + 1$

$= x^2 + 8x + 1$

10. $f(a) = -3\sqrt{a^2 + 9}$

a. $f(4) = -3\sqrt{4^2 + 9}$

$= -3\sqrt{25}$

$= -3 \cdot 5 = -15$

b. $f(3a) = -3\sqrt{(3a)^2 + 9}$

$= -3\sqrt{9a^2 + 9}$

$= -3\sqrt{9(a^2 + 1)} = -3 \cdot 3\sqrt{a^2 + 1}$

$= -9\sqrt{a^2 + 1}$

c. $f(a+1)$

$= -3\sqrt{(a+1)^2 + 9}$

$= -3\sqrt{a^2 + 2a + 1 + 9}$

$= -3\sqrt{a^2 + 2a + 10}$

For #11-12, state the domain of each function.

11. $g(x) = \sqrt{-3x - 2}$

$-3x - 2 \geq 0$

$-3x \geq 2$

$x \leq -\frac{2}{3}$ or $(-\infty, -\frac{2}{3}]$

12. $h(t) = \dfrac{2t - 6}{t^2 + 6t + 9}$

$(t+3)(t+3)$

All $\mathbb{R}$ except $t \neq -3$

13. Find f(-4) and f(11) for the piecewise function $f(x)=\begin{cases}3x^2+16 \text{ if } x<-2\\ \sqrt{x-2} \text{ if } -2<x\le 11\\ -75 \text{ if } x>11\end{cases}$

$f(-4)=3(-4)^2+16=64$

$f(11)=\sqrt{11-9}=\sqrt{9}=3$

14. An elevator starts with 12 people on a building's 8th floor. One person exits to each floor. The lowest level is two floors below ground level. The function $f(l)=l+4$ gives the number of people on the elevator after a person exits to that level.

l - Building level

$f(l)$ - # of people on the elevator

a. Write the relevant domain in set-builder notation.

$D=\{l \mid -2\le l\le 8,\ l\in\mathbb{Z}\}$

b. Write the range in set-builder notation.

$R=\{f(l) \mid 2\le f(l)\le 12,\ f(l)\in\mathbb{Z}\}$

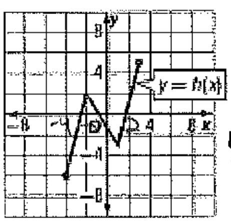
$f(-2)=2$

$f(8)=12$

Section 1-2 Content

For #15-16, use the graph of h to find the domain and range of each function.

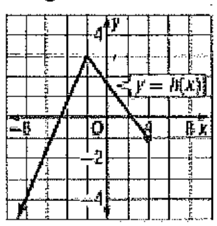
15.



D: $[-4,3]$

R: $[-6,5]$

16.



D: $(-\infty,4]$

R: $(-\infty,3]$

17. Use the graph of the function to find its y-intercept and zeros. Then find these values algebraically.

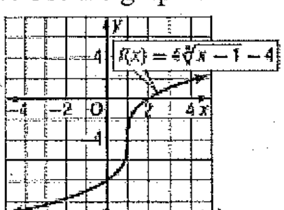
Y-int:

$4\sqrt[3]{0-1}-4$

$4(-1)-4$

$-4-4$

$=-8$



Y-int: $(0,-8)$

zeros: $(2,0)$

Y-int algebraically:

$Y=4\sqrt[3]{0-1}-4=4(-1)-4=-8$

zero algebraically:

$0=4\sqrt[3]{x-1}-4$

$4=4\sqrt[3]{x-1}$

$1=\sqrt[3]{x-1}$

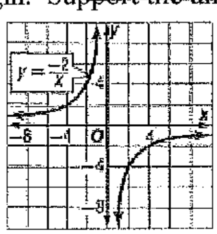
$1^3=x-1$

$2=x$

x-int: $0=4\sqrt[3]{x-1}$ $4=4\sqrt[3]{x-1}\to 1=\sqrt[3]{x-1}$

For #18-19, Use the graph of each equation to test for symmetry with respect to the x-axis and y-axis, and the origin. ~~Support the answer numerically and~~ confirm algebraically.

18.



Sym. about origin

$\therefore$ odd fn

$Y=-\frac{2}{X}$

$-Y=-\frac{2}{-X}$

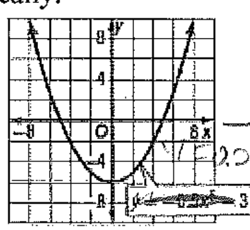
$-Y=\frac{2}{X}$

$Y=-\frac{2}{X}$

Same!

If it's symmetric over the origin for every point (x,y) we have $(-x,-y)$

19.



Sym. about Y-axis

$\therefore$ even fn

$Y=0.25x^2-6$

$Y=0.25(-x)^2-6$

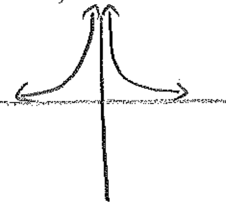
$Y=0.25x^2-6$

Same

If it's symmetric over the Y-axis for every point (x,y) we have $(-x,y)$.

VA: x=0   HA: y=0

20. Graph $g(x)=\dfrac{1}{x^2}$ using a graphing calculator. Analyze the graph to determine whether the function is even, odd, or neither. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.

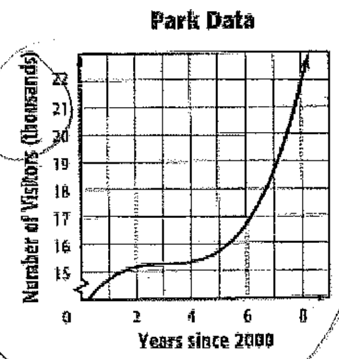


Reflects over y-axis ∴ even

If even $f(-x)=f(x)$

$\dfrac{1}{(-x)^2}=\dfrac{1}{x^2}\,? \rightarrow \dfrac{1}{x^2}=\dfrac{1}{x^2}$ ✓

21. The approximate numbers of annual visitors to a park from 2000 through 2008 can be modeled using $v(x)=0.05x^3-0.51x^2+1.81x+13.35$, where x represents the number of years since year 2000.



a. Use the graph to estimate the number of visitors to the park in 2006.

≈ 16,700 visitors

b. Find the estimated number of visitors in 2006 algebraically.

$V(6)=0.05(6)^3-0.51(6)^2+1.81(6)+13.35$
$=16{,}650$ (multiply ans. by 1,000)

c. In what year did the number of visitors first exceed 20,000?

$\underset{Y_1}{20}=\underset{Y_2}{0.05x^3-0.51x^2+1.81x+13.35}$

Do 2nd → Calc → Intersect

$x=7.74$ ∴ During year 2007

Section 1-3 Content

For # 22-25, determine whether each function is continuous at the given x-value(s). Justify using the continuity test. If discontinuous identify the type of discontinuity.

22. $f(x)=-\dfrac{2}{3x^2}$; *at* $x=-1$

$f(-1)=\lim\limits_{x\to -1^-} f(x)=\lim\limits_{x\to -1^+} f(x)\,?$

$-\frac{2}{3}=-\frac{2}{3}=-\frac{2}{3}$

∴ Continuous

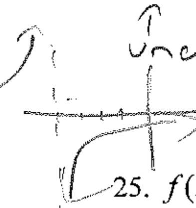
23. $f(x)=\dfrac{x-2}{x+4}$; *at* $x=-4$

$\lim\limits_{x\to -4^-} f(x)\neq\lim\limits_{x\to -4^+} f(x)$

$f(-4)=\lim\limits_{x\to -4^-} f(x)=\lim\limits_{x\to -4^+} f(x)\,?$
$\qquad\quad +\infty \qquad\qquad -\infty$

und. ∴ Not continuous

Infinite Discontinuity



24. $f(x)=x^3-2x+2$; *at* $x=1$

$f(1)=\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^+} f(x)\,?$

$1=1=1$

∴ Continuous

25. $f(x)=\dfrac{x+1}{\cancel{x^2+3x+2}}$; *at* $x=-1$ *and* $x=-2$

$(x+2)(x+1)$

f(x) is undefined for x = -1 and -2

∴ Not continuous at either.

Removable Discontinuity @ x = -1 | Infinite Discon. @ x = -2

26. Determine between which consecutive integers the real zeros of each function are located on the given interval.

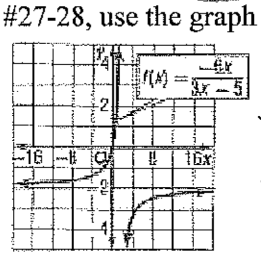
$f(x)=x^3+5x^2-4;\ [-6,+2]$

X	-6	-5	-4	-3	-2	-1	0	1	2
Y	-40	-4	12	14	8	0	-4	2	24

[-5, -4], [-1, 0], [0, 1]

For #27-28, use the graph of each function to describe its end behavior.

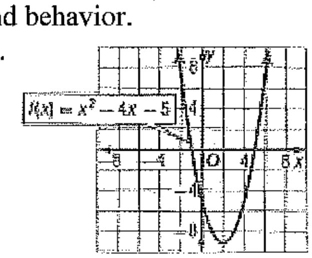
27.



$\lim\limits_{x\to -\infty} f(x)=-2^+$

$\lim\limits_{x\to \infty} f(x)=-2^-$

28.



$\lim\limits_{x\to -\infty} f(x)=\infty$

$\lim\limits_{x\to \infty} f(x)=\infty$

29. The per-person cost of a guided climbing expedition can be modeled by $f(x) = \frac{600}{x+25}$, where x is the number of people on the trip.

a. Graph the function using a graphing calculator. Use the graph to identify and describe any points of discontinuity.

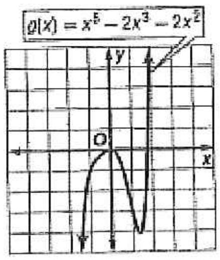
Infinite Discontinuity @ X=-25

b. Are there any points of discontinuity in the relevant domain? Explain.

NO. X will not be negative since X represents an amount of people.

Section 1-4 Content

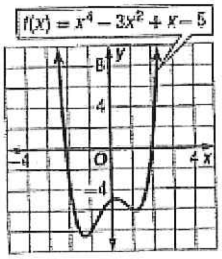
30 ~~29.~~ Use the graph of the given function to estimate the intervals on which the function is increasing, decreasing, or constant.



Inc: $(-\infty, 0) \cup (1.25, \infty)$

Dec: $(0, 1.25)$

31. ~~30.~~ Estimate and classify the extrema for the graph of the given function.



Abs. min. @ $(-1.3, -8.5)$

Rel. min. @ $(1.25, -6)$

Rel. max. @ $(0.25, -5)$

30. ~~31.~~ Find the average rate of change (average speed) of each function on the given interval.

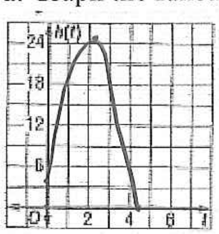
$g(x) = x^4 + 2x^2 - 5$; $[-4, -2]$ $f(-4) = 283$ $f(-2) = 19$

$(-4, 283)$

$(-2, 19)$ A.R.C. (m) $= \frac{(19-283)}{(-2 - -4)} = -132$

32. ~~32.~~ A lost boater shoots a flare straight up into the air. The height of the flare, in meters, can be modeled by $h(t) = -4.9t^2 + 20t + 4$, where t is the time in seconds since the flare was launched.

a. Graph the function.



b. Estimate the greatest height reached by the flare.

24.40 m.

Y-int: $(0, 4)$

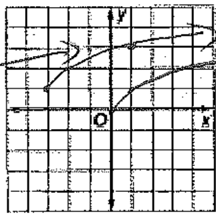
X-int: $(4.27, 0)$

max: $(2.04, 24.40)$

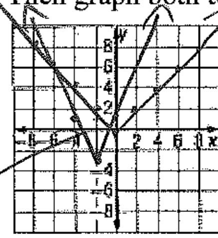
L2, D3 scale factor of 2 vertically

Section 1-5 Content

~~33~~ 34. Use the graph of $f(x)=\sqrt{x}$ to graph $g(x)=\sqrt{x+3}+1$. L 3, U 1



~~34~~ 35. Identify the parent function $f(x)$ of $g(x)=2|x+2|-3$. Describe how the graphs are related. Then graph both together.



Section 1-6 Content

For #~~35-36~~ 36-37, find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$. State the domain of the new function.

~~35~~ 36. $f(x)=2x^2+8$ and $g(x)=5x-6$ (All IR, All IR)

$(f+g)(x)=2x^2+8+5x-6=2x^2+5x+2$
D: All IR

$(f-g)(x)=2x^2+8-(5x-6)=2x^2-5x+14$
D: All IR

$(f\cdot g)(x)=(2x^2+8)(5x-6)=10x^3-12x^2+40x-48$
D: All IR

$\left(\frac{f}{g}\right)(x)=2x^2+8/5x-6$ D: All IR except $x\neq 6/5$

~~36~~ 37. $f(x)=x^3$ and $g(x)=\sqrt{x+1}$ (All IR; D: $x+1\geq 0$, $x\geq -1$)

$(f+g)(x)=x^3+\sqrt{x+1}$ D: $[-1,\infty)$

$(f-g)(x)=x^3-\sqrt{x+1}$ D: $[-1,\infty)$

$(f\cdot g)(x)=x^3\sqrt{x+1}$ D: $[-1,\infty)$

$\left(\frac{f}{g}\right)(x)=\frac{x^3}{\sqrt{x+1}}=\frac{x^3\sqrt{x+1}}{x+1}$ D: $(-1,\infty)$

For #~~37-38~~ 38-39, find $[f\circ g](x)$, $[g\circ f](x)$, and $[f\circ g](3)$.

~~37~~ 38. $f(x)=2x^3-3x^2+1$ and $g(x)=3x$

$f(g(x))=2(3x)^3-3(3x)^2+1$
$=2\cdot 27x^3-3\cdot 9x^2+1$
$=54x^3-27x^2+1$

$g(f(x))=3(2x^3-3x^2+1)=6x^3-9x^2+3$

$f(g(3))=54(3)^3-27(3)^2+1=1216$

~~38~~ 39. $f(x)=x+5$ and $g(x)=x-3$

$f(g(x))=x-3+5=x+2$

$g(f(x))=x+5-3=x+2$

$f(g(3))=3+2=5$

For #~~39-40~~ 40-41, find $f\circ g$.

~~39~~ 40. $f(x)=\sqrt{x-2}$
$g(x)=3x$

$f(g(x))=\sqrt{3x-2}$

~~40~~ 41. $f(x)=\frac{1}{x-8}$
$g(x)=x^2+5$

$f(g(x))=\frac{1}{x^2+5-8}$
$=\frac{1}{x^2-3}$

For #~~41-42~~ 42-43, find two functions f and g such that $h(x)=[f\circ g](x)$.

~~41~~ 42. $h(x)=\sqrt{2x-6}-1$

$f(g(x))=\sqrt{2x-6}-1$
$f(x)=\sqrt{x}-1$
$g(x)=2x-6$

~~42~~ 43. $h(x)=\frac{1}{3x+3}$

$f(g(x))=\frac{1}{3x+3}$
$f(x)=\frac{1}{x}$
$g(x)=3x+3$

Section 1-7 Content

For #43-44, determine whether f has an inverse function. If it does, find the inverse function and state any restrictions to the domain.

44 43. $f(x) = \sqrt[3]{x-1}$ ← yes, passes HLT

$x = \sqrt[3]{y-1}$

$x^3 = y-1$

$x^3+1 = y$

$f^{-1}(x) = x^3-1$

46 D: All ℝ

45 44. $f(x) = \dfrac{2x-1}{x+7}$ ← yes, passes HLT

$x = \dfrac{2y-1}{y+7}$

$x(y+7) = 2y-1$

$xy+7x = 2y-1$

$xy-2y = -7x-1$

$y(x-2) = -7x-1$

$y = \dfrac{-7x-1}{x-2}$

D: $x \neq 2$

45. Show algebraically that f and g are inverse functions.

$f(x) = 2x+3;\ g(x) = \dfrac{x-3}{2}$

$f(g(x)) = \cancel{2}\left(\dfrac{x-3}{\cancel{2}}\right)+3$

$= x - \cancel{3} + \cancel{3}$

$= x$ ✓

$g(f(x)) = \dfrac{2x+\cancel{3}-\cancel{3}}{2}$

$= \dfrac{\cancel{2}x}{\cancel{2}}$

$= x$ ✓

show graph too

47 46. Use the graph of $f(x)$ to graph $f^{-1}(x)$