

Addition & Subtraction of Rational Expressions

Honors Algebra 2

Notes Section 8.5 (#9)

Name Key 2014-15

Hour _____ Date _____

Lesson Objective: Students will be able to add & subtract rational expressions.

Warm Up: Complete the following problems without a calculator.

OTO

a. $\frac{1}{7} + \frac{3}{7} = \frac{4}{7}$

b. $\frac{4}{15} + \frac{2}{5} = \frac{4}{15} + \frac{6}{15} = \frac{10}{15} = \frac{2}{3}$

c. $\frac{3}{4} - \frac{1}{7} = \frac{21}{28} - \frac{4}{28} = \frac{17}{28}$

Key Ideas:

Adding or Subtracting with Like (common) Denominators

- To add (or subtract) rational expressions with LIKE denominators, simply add (or subtract) the numerators, and place over the common denominator.
- Always write resultant fraction **is** simplified form if possible.

Ex. $\frac{4x}{x-2} - \frac{x}{x-2} = \frac{3x}{x-2}, x \neq 2$

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ where $c \neq 0$

Adding or Subtracting with Unlike (different) Denominators

- To add (or subtract) rational expressions with DIFFERENT denominators, first rewrite each fraction with a common denominator. Then simply add (or subtract) the numerators, and place over the common denominator. *Find LCM*

LCM = 28x

$\frac{3}{4x} - \frac{3}{7} = \frac{21}{28x} - \frac{12x}{28x} = \frac{21-12x}{28x} = \frac{3(7-4x)}{28x}$

↑ Try to simplify!

- The best practice to finding a common denominator is to find the LCM (least common multiple of the denominators). This helps minimize the effort needed to reduce the resultant fraction.

LCM - The smallest multiple of the common multiples.

- Always write resultant fraction **is** simplified form if possible.

1) what is the LCM of 6 and 15?

multiples of 6: 6, 12, 18, 24, 30, ...

multiples of 15: 15, 30, 45, 60, ...

∴ LCM = 30

what is the LCM of $6ab$ and $8ab^2$
 $LCM = 24ab^2$

LCM Product of
 For : each factor
 variables that has
 the highest
 power.

Ex. 1] Perform the indicated operation and simplify.

a. $\frac{4}{2x} + \frac{3}{2x}$

$= \frac{7}{2x}, x \neq 0$

b. $\frac{3}{2x^2} + \frac{2}{3x}$ LCM: $6x^2$

$= \frac{9}{6x^2} + \frac{4x}{6x^2}$
 $= \frac{4x+9}{6x^2}, x \neq 0$

c. $\frac{x}{x+4} - \frac{3}{x+4}$

$= \frac{x-3}{x+4}, x \neq -4$

f. $\frac{3x}{16x^2-8x} + \frac{x}{16x^2-8x}$

$= \frac{4x}{16x^2-8x}$
 $= \frac{4x}{8x(2x-1)}$
 $= \frac{1}{2(2x-1)}, x \neq 0, \frac{1}{2}$

g. $\frac{2x}{3y} - \frac{5}{3y}$

$= \frac{2x-5}{3y}$
 $= \frac{2x-5y}{3y}, y \neq 0$

h. $\frac{4}{\frac{2}{x} + \frac{1}{x}}$

$= \frac{4}{\frac{3}{x}} = \frac{4x}{3}, x \neq -\frac{1}{2}, 0$

d. $\frac{7}{x^2-4} - \frac{2x}{x+2}$ LCM: $(x+2)(x-2)$

$= \frac{7}{(x+2)(x-2)} - \frac{2x}{x+2}$
 $= \frac{7}{(x+2)(x-2)} - \frac{2x(x-2)}{(x+2)(x-2)}$
 $= \frac{7 - 2x^2 + 4x}{(x+2)(x-2)}, x \neq \pm 2$

e. $\frac{-16x+20}{3x^2-12x-15} + \frac{2x}{3x-15}$ LCM: $3(x-5)(x+1)$

$= \frac{-16x+20}{3(x^2-4x-5)} + \frac{2x}{3(x-5)}$
 $= \frac{-16x+20}{3(x-5)(x+1)} + \frac{2x}{3(x-5)}$
 $= \frac{-16x+20}{3(x-5)(x+1)} + \frac{2x(x+1)}{3(x-5)(x+1)}$
 $= \frac{-16x+20+2x^2+2x}{3(x-5)(x+1)}$
 $= \frac{2x^2-14x+20}{3(x-5)(x+1)}$
 $= \frac{2(x^2-7x+10)}{3(x-5)(x+1)}$
 $= \frac{2(x-5)(x-2)}{3(x-5)(x+1)}$
 $= \frac{2(x-2)}{3(x+1)}, x \neq 5, -1$

#9 Homework:

Pg. 586-588
 (4-8, 17-23, 28)

Pg. 605
 (19-22)



section 8.5 P. 586-588

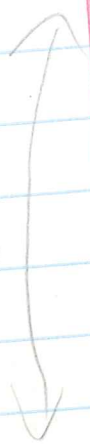
$$(4) \quad \frac{x}{16x^2} - \frac{4}{16x^2} = \boxed{\frac{(x-4)}{16x^2}, x \neq 0}$$

$$(5) \quad \frac{9}{x+1} - \frac{2x}{x+1} = \boxed{\frac{9-2x}{x+1}, x \neq -1}$$

$$(7) \quad \frac{5x}{x+3} + \frac{15}{x+3} = \frac{5x+15}{x+3}$$

$$= \frac{5(x+3)}{(x+3)}$$

$$= \boxed{5, x \neq -3}$$



$$(6) \quad \frac{3x^2}{x-8} + \frac{6x}{x-8} = \frac{3x^2+6x}{(x-8)}$$

$$= \boxed{\frac{3x(x+2)}{(x-8)}, x \neq 8}$$

$$(8) \quad \frac{4x^2}{2x-1} - \frac{1}{2x-1} = \frac{4x^2-1}{(2x-1)}$$

$$= \frac{(2x+1)(2x-1)}{(2x-1)} = \boxed{(2x+1), x \neq \frac{1}{2}}$$

$$\text{LCD} = 12x^2$$

$$\begin{aligned} \textcircled{17} \quad \frac{8}{3x^2} - \frac{5}{4x} &= \frac{4 \cdot 8}{4 \cdot 3x^2} - \frac{5 \cdot 3x}{4x \cdot 3x} \\ &= \frac{32}{12x^2} - \frac{15x}{12x^2} \end{aligned}$$

$$\boxed{= \frac{32 - 15x}{12x^2}, x \neq 0}$$

$$\textcircled{18} \quad \frac{x-4}{5x} - \frac{12}{5(x-4)} \quad \text{LCD} = 5x(x-4)$$

$$= \frac{(x-4) \cdot (x-4)}{(x-4) \cdot 5x} - \frac{12 \cdot x}{5(x-4) \cdot x}$$

$$= \frac{\overbrace{(x-4)(x-4)} - \frac{12x}{5x(x-4)}}{5x(x-4)}$$

$$= \frac{x^2 - 4x - 4x + 16 - 12x}{5x(x-4)}$$

$$\boxed{= \frac{x^2 - 20x + 16}{5x(x-4)}, x \neq 0, 4}$$

$$\begin{array}{r} 16x^2 \\ 7 \quad \times \quad 7 \\ \hline -20x \end{array}$$

$$\text{LCD} = (x+8)(x-3)$$

$$(19) \frac{12}{x^2+5x-24} + \frac{3}{(x-3)}$$

$$= \frac{12}{(x+8)(x-3)} + \frac{3}{(x-3)}$$

$$= \frac{12}{(x+8)(x-3)} + \frac{3 \cdot (x+8)}{(x-3)(x+8)}$$

$$= \frac{12}{(x+8)(x-3)} + \frac{3x+24}{(x+8)(x-3)}$$

$$= \frac{12+3x+24}{(x+8)(x-3)}$$

$$= \frac{3x+36}{(x+8)(x-3)}$$

$$= \frac{3(x+12)}{(x+8)(x-3)}, x \neq -8, 3$$

$$\text{LCD} = (x+4)(x+6)$$

$$\textcircled{20} \quad \frac{3}{(x+4)} - \frac{1}{(x+6)}$$

$$= \frac{(x+6) \cdot 3}{(x+6)(x+4)} - \frac{1 \cdot (x+4)}{(x+6)(x+4)}$$

$$= \frac{3(x+6)}{(x+6)(x+4)} - \frac{(x+4)}{(x+6)(x+4)}$$

$$= \frac{3x+18-x-4}{(x+6)(x+4)}$$

$$= \frac{2x+14}{(x+6)(x+4)}$$

$$= \frac{2(x+7)}{(x+6)(x+4)} \quad | \quad x \neq -6, -4$$

$$\text{LCD} = (x-3)(x+1)$$

$$\textcircled{21} \text{ (x+1)} \cdot \frac{9}{(x-3)} + \frac{2x}{(x+1)} \cdot \frac{(x-3)}{(x-3)}$$

$$= \frac{9(x+1)}{(x+1)(x-3)} + \frac{2x(x-3)}{(x+1)(x-3)}$$

$$= \frac{9x+9+2x^2-6x}{(x+1)(x-3)}$$

$$\begin{array}{r} 18x^2 \\ \cancel{2x} \\ \hline 3x \end{array}$$

$$= \frac{2x^2 + 3x + 9}{(x+1)(x-3)}, \quad x \neq -1, 3$$

$$\textcircled{22} \quad \frac{x+4}{x^2-4} = \frac{15}{x-2}$$

$$\text{LCD} = (x-2)(x+2)$$

$$= \frac{(x+4)}{(x+2)(x-2)} - \frac{15 \cdot (x+2)}{(x-2)(x+2)}$$

$$= \frac{-2(7x+13)}{(x+2)(x-2)}$$
$$x \neq \pm 2$$

$$= \frac{x+4-15x-30}{(x+2)(x-2)}$$

$$= \frac{-14x-26}{(x+2)(x-2)}$$

$$\text{LCD} = (x-4)(x-4)$$

$$\textcircled{23} \quad \frac{-15x}{x^2-8x+16} + \frac{12}{(x-4)}$$

$$= \frac{-15x}{(x-4)(x-4)} + \frac{12}{(x-4)} \cdot \frac{(x-4)}{(x-4)}$$

$$= \frac{-15x + 12x - 48}{(x-4)(x-4)}$$

$$= \frac{-3x - 48}{(x-4)(x-4)}$$

$$\boxed{= -\frac{3(x+16)}{(x-4)(x-4)}, x \neq 4}$$

$$\textcircled{24} \quad \frac{1}{x^2-8} - \frac{1}{(x-2)}$$

$$= \frac{1}{(x+2)(x-2)} - \frac{1}{(x-2)}$$

$$= \frac{1}{(x+2)(x-2)} - \frac{1}{(x-2)}$$

$$LCD = (x-4)(x+2)(x-8)$$

$$\textcircled{28} \quad \frac{(x+3)}{x^2-2x-8} - \frac{(x-5)}{x^2-12x+32}$$

$$\frac{(x-8)}{(x-8)} \cdot \frac{(x+3)}{(x-4)(x+2)} - \frac{(x-5)}{(x-8)(x-4)} \cdot \frac{(x+2)}{(x+2)}$$

$$= \frac{(x-8)(x+3) - (x-5)(x+2)}{(x-4)(x+2)(x-8)}$$

$$= \frac{x^2+3x-8x-24 - (x^2+2x-5x-10)}{(x-4)(x+2)(x-8)}$$

$$= \frac{\cancel{x^2} - 5x - 24 - (\cancel{x^2} - 3x - 10)}{(x-4)(x+2)(x-8)}$$

$$= \frac{-2x-14}{(x-4)(x+2)(x-8)}$$

$$\boxed{= \frac{-2(x+7)}{(x-4)(x+2)(x-8)}, x \neq 4, -2, 8}$$

P. 605 # 19-22

(19)

$$\frac{80x^4}{y^3} \cdot \frac{xy}{5x^2} = \frac{80x^3y}{5x^2y^3}$$

$$= \frac{16x^3}{y^2}, x \neq 0, y \neq 0$$

(20)

$$\frac{x-3}{2x-8} \cdot \frac{6x^2-96}{x^2-9}$$

$$= \frac{(x-3)}{2(x-4)} \cdot \frac{6(x^2-16)}{(x+3)(x-3)}$$

$$= \frac{(x-3)}{2(x-4)} \cdot \frac{6(x+4)(x-4)}{(x+3)(x-3)}$$

$$= \frac{6(x-3)(x+4)(x-4)}{2(x-4)(x+3)(x-3)}$$

$$= \frac{3(x+4)}{(x+3)}, x \neq 4, -3, 3$$

$$\begin{array}{r} 16x^2 \\ -4x \quad -4x \\ \hline -8x \end{array}$$

$$\begin{aligned} & (16x^2 - 4x) + (-4x + 1) \\ &= 4x(4x - 1) - 1(4x - 1) \\ &= (4x - 1)(4x - 1) \end{aligned}$$

$$\textcircled{21} \quad \frac{16x^2 - 8x + 1}{x^3 - 7x^2 + 12x} \div \frac{20x^2 - 5x}{15x^3}$$

$$\frac{(4x-1)(4x-1)}{x(x^2-7x+12)} \div \frac{5x(4x-1)}{15x^3}$$

$$= \frac{(4x-1)(4x-1)}{x(x-3)(x-4)} \cdot \frac{15x^3}{5x(4x-1)}$$

$$= \frac{15x^{\cancel{3}}(4x-1)(4x-1)}{5x^{\cancel{2}}(x-3)(x-4)(4x-1)}$$

$$= \frac{3x(4x-1)}{(x-3)(x-4)}$$

(22)

$$\frac{x^2 - 13x + 40}{x^2 - 2x - 15} \div \frac{x^2 - 5x - 24}{1}$$

$$= \frac{(x-8)(x-5)}{(x-5)(x+3)} \cdot \frac{1}{(x-8)(x+3)}$$

$$= \frac{\cancel{(x-8)}\cancel{(x-5)}}{\cancel{(x-5)}(x+3)\cancel{(x-8)}(x+3)}$$

$$= \frac{1}{(x+3)(x+3)}, x \neq 5, -3, 8, -3$$