

Sequences & Series

For question #1 & 2, find the first four terms of the sequence described:

1. $a_1 = 5, r = 3$

$$\boxed{5, 15, 45}$$

2. $a_1 = 2, d = 6$

$$\boxed{2, 8, 14}$$

For question #3-6, find the sum of the given series:

3. $\sum_{k=2}^6 3(2)^{k-1}$

$$= 3(2)^1 + 3(2)^2 + 3(2)^3 + 3(2)^4 + 3(2)^5 \\ = 6 + 12 + 24 + 48 + 96 \\ = \boxed{186}$$

4. $4 + 11 + 18 + 25 + \dots n = 10$

$$S_{10} = 10 \left(\frac{4+67}{2} \right) \\ = \boxed{355}$$

5. $1 + 2 + 4 + 8 + 16 + \dots n = 20$

$$S_{20} = 1 \cdot \left(\frac{2^{20} - 1}{2 - 1} \right) \\ = \boxed{1,048,575}$$

6. $\sum_{k=1}^4 7k - 3$

$$\left\{ \begin{array}{l} a_n = 7n - 3 \\ a_{10} = 7(10) - 3 \\ a_{10} = 70 - 3 = 67 \end{array} \right. \\ \left\{ \begin{array}{l} = [7(1) - 3] + [7(2) - 3] + [7(3) - 3] + [7(4) - 3] \\ = 4 + 11 + 18 + 25 \\ = \boxed{58} \end{array} \right.$$

For question #7-10, find the specific term in the sequence.

7. a_{19} in the sequence of $\frac{1}{2}, 3, 18, 108, \dots$

$$a_{19} = \left(\frac{1}{2}\right) \cdot 6^{18}$$

$$\boxed{a_{19} = 5.07 \times 10^{13}}$$

8. a_{17} in the sequence of 11, 20, 29, ...

$$a_{17} = 9(17) + 2$$

$$\boxed{a_{17} = 155}$$

9. $a_1 = 20, d = 3, a_{20} = ?$

$$a_{20} = 20 + (19)(3)$$

$$\boxed{a_{20} = 77}$$

10. $a_1 = -5, r = 2, a_{12} = ?$

$$a_{12} = (-5)(2^{11})$$

$$\boxed{a_{12} = -10,240}$$

11. Find the missing terms in the arithmetic sequence: 16, 10, 4, -2, -8

$$-8 = 16 + (5-1)d$$

$$-24 = 4d$$

$$d = -6$$

12. Find the missing terms in the geometric sequence: 7, 14, 28, 56, 112

$$112 = 7 \cdot r^{5-1}$$

$$\sqrt[4]{112} = \sqrt[4]{r^4}$$

$$r = 2$$

For question #13 & 14, write an explicit equation for the n^{th} term of the sequence.

13. 3, 24, 192, ...

14. -13, 1, 15, ...

$$a_n = 3 \cdot 8^{n-1}$$

$$a_n = 14n - 27$$

or

$$a_n = -13 + 14(n-1)$$

For question #15 & 16, identify where the term falls in the given sequence.

15. Complete the statement: 117 is the 76th term of -33, -31, -29, -27, ...

$$117 = -33 + (n-1)(2)$$

$$150 = 2(n-1)$$

$$75 = n-1$$

$$\boxed{76 = n}$$

16. Complete the statement: 118,098 is the 10th term of 6, 18, 54, 162, ...

$$118,098 = 6 \cdot 3^{n-1}$$

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$$19683 = 3^{n-1}$$

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$$3^9 = 3^{n-1}$$

$$\log 19683 = \log 3^{n-1}$$

$$9 = n-1$$

$$\boxed{n=10}$$

OR

$$\frac{\log 19683}{\log 3} = \frac{(n-1) \log 3}{\log 3}$$

$$9 = n-1$$

$$\boxed{n=10}$$

17. Find the sum of the first 50 positive multiples of 5.

$$5 + 10 + 15 + 20 + \dots$$

$$a_{50} = 5(50) \quad S_{50} = 50 \left(\frac{5+250}{2} \right)$$

$$a_{50} = 250$$

$$S_{50} = \boxed{6375}$$

18. Write a recursive rule for the following sequence: 16, 26, 36, 46, ...

$$a_1 = 16$$

$$a_n = a_{n-1} + 10$$