Review – Section 7.1, 7.2 & Compound Interest Name Key 2015-16
Honors Algebra 2

$A = P\left(1+\frac{r}{n}\right)^{nt}$ can be used to calculate the yield of an investment that is compounded: annually, semiannually, quarterly, monthly, weekly, daily, hourly, or every minute.

A-value of investment in the future, P-original investment, r-interest rate, t-# of years of investing, n-# of times interest is compounded in a year

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$A = Pe^{rt}$ can be used to calculate the yield of an investment that is compounded continuously.

A-value of the investment in the future, P-original investment, r-interest rate, t-# of years of investing.

$$A = P\left[\frac{(1+r)^t - 1}{r}\right](1+r)$$

can be used to calculate the yield of an investment while deposits are still being made.

A-value of the investment in the future, r-interest rate, t-# of periods, P-deposit amount.

Use the following scenario to answer question #1-4:

Suppose you deposit money into a savings program that earns 9% interest compounded annually.

1. If you deposit $1,750 every year for 20 years, how much money will you have after 20 years?

$$A = 1750\left[\frac{(1+0.09)^{20}-1}{0.09}\right](1+0.09) = \$97,587.93$$

2. Suppose you leave the money in the account for another 15 years after you stop making deposits. How much money would you have?

$$A = 97,587.93\left(1+\frac{0.09}{1}\right)^{1\cdot 15} = \$355,462.32$$

3. How much longer would you have to keep the money in the account to have over $500,000?

$$\underbrace{500,000}_{f_1} = \underbrace{355,462.32(1.09)^t}_{f_2}$$

$t \approx 3.96$ years

4. Suppose you continued to deposit $1,750 for all 35 years. How much more money would you have made if you continued making deposits?

$$A = 1750\left[\frac{(1+0.09)^{35}-1}{0.09}\right](1.09) = \$411,468.26$$

How much more $= 411,468.26 - 355,462.32 = \$56,005.94$

5. How much money would you make if you invest $12,000 in a bank for 40 years at an interest rate of 7.5% compounded quarterly?

$$A = 12{,}000\left(1 + \frac{0.075}{4}\right)^{4 \cdot 40}$$

$$= \$234{,}423.27$$

6. How much money would you make if you invested $750 in a bank account for 37 years that compounds interest continuously at a rate of 11%?

$$A = 750e^{0.11 \cdot 37}$$

$$= \$43{,}917.72$$

7. How long would it take to double an investment of $2,000 at 5% interest compounded monthly?

$$\underbrace{4{,}000}_{f_1} = \underbrace{2{,}000\left(1 + \frac{0.05}{12}\right)^{12t}}_{f_2}$$

$$t \approx 13.9 \text{ years}$$

8. You put your money in a bank account that compounds the interest daily. What interest rate would be needed to turn $300 into $1,466 in fourteen years?

$$1{,}466 = 300\left(1 + \frac{r}{365}\right)^{365 \cdot 14}$$

$$1{,}466 = 300\left(1 + \frac{r}{365}\right)^{5{,}110}$$

$$\frac{1{,}466}{300} = \left(1 + \frac{r}{365}\right)^{5{,}110}$$

$$\sqrt[5{,}110]{\frac{1{,}466}{300}} = 1 + \frac{r}{365}$$

$$\left(\sqrt[5{,}110]{\frac{1{,}466}{300}} - 1\right) \cdot 365 = r$$

$$0.113 = r$$

or 11.3%

9. You are saving to buy a new car that is worth $25,000. You currently have $5,000. How long would it take to save enough money to buy the car if you put your money into a bank account that earns 14% interest compounded weekly?

$$\underbrace{25{,}000}_{f_1} = \underbrace{5{,}000\left(1 + \frac{0.14}{52}\right)^{52t}}_{f_2}$$

$$\approx 11.5 \text{ years}$$

10. The total population of Ann Arbor, Michigan has grown exponentially over time. The table below displays the Ann Arbor population recorded in the census from 1970 until 2000.

Population of Ann Arbor			
Year	Total Population (*)	Central Area Population (**)	Central Area % of Total Population (***)
1970	99,797	36,000	36
1980	107,966	34,781	32
1990	109,592	36,058	33
2000	114,024	33,550	29

Source:
http://www.a2gov.org/government/publicservices/systems_planning/Environment/soe07/healthyurban/Pages/PopulationDensity.aspx

a. Write an equation to model the **total** population of Ann Arbor. Use the total population for year 1970 and year 2000. Be sure to verify that the equation works by checking populations for year 1970 and 2000.

$Y = a(b)^x$

$114,024 = 99,797(b)^{30}$

$\frac{114,024}{99,797} = b^{30}$

$\sqrt[3]{\frac{114,024}{99,797}} = b$

$b = 1.00445243$

$Y = 99,797(1.00445243)^x$

X - # of years after 1970

Y - Population of Ann Arbor

b. Use your model to predict the **total** population of Ann Arbor in year 2025.

$Y = 99,797(1.00445243)^{2025-1970}$ $\approx 127,417$ people

c. When will the Ann Arbor's **total** population exceed 135,000 people?

$135,000 = 99,797(1.00445243)^x$

$x \approx 68$ years later or year 2038

d. Estimate Ann Arbor's **total** population when the next census is taken.

$Y = 99,797(1.00445243)^{2020-1970}$

$\approx 124,618$ people

2013

11. Paul went out at midnight on Friday, November 15th to get his Playstation 4 for $399. Unfortunately, the value of his Playstation 4 will decline exponentially over time. The value of the PS4 will decline 15.75% every year. $b = \frac{(100-15.75)}{100} = 0.8425$

a. Write an equation to model this scenario.

$Y = 399(0.8425)^x$

b. How much will his PS4 be worth in 10 years?

$Y = 399(0.8425)^{10} = \$71.89$

c. When will the PS4 be worth less than $50.

$50 = 399(0.8425)^x$ $\quad x \approx 12\text{-}13$ years later

$f_1 \quad f_2$

d. In the year 2021 Paul will debate if he should trade in his PS4 to get the PS5. If he wants to get at least $125 for it should he trade it in? Why?

$Y = 399(0.8425)^{2021-2013}$

$= \$101.28$

No Deal! He would only get $101.28 for it.

For #12-13, find an exponential model for each.

12. The function contains the points (0, 3) and (3, 1029)

X	Y
0	3
3	1029

$Y = a(b)^x$

$1029 = 3(b)^3$

$\frac{1029}{3} = b^3$

$\sqrt[3]{\frac{1029}{3}} = b$

$7 = b$

$Y = 3(7)^x$

13. The function contains the points (2, 72) and (6, 93312)

X	Y
0	2
1	12
2	72
3	
4	
5	
6	93,312

$Y = a(b)^x$

$93{,}312 = 72(b)^4$

$\frac{93{,}312}{72} = b^4$

$\sqrt[4]{\frac{93{,}312}{72}} = b$

$6 = b$

$Y = 2(6)^x$

Determine whether each function represents exponential growth or decay.

14. $y = 0.75(5)^x$

Growth

15. $y = \left(\frac{3}{4}\right)^x$

Decay

16. $y = 3(0.90)^x$

Decay