

Section 1.1: Choosing a Cell Phone Plan

Isabelle Nueva is trying to help her mother and father decide on the best cell phone plan for her family. She and her friend, Angelo Franco, think that some of the things they have learned in their math class about making decisions will be helpful to them. They studied a process for making decisions that have multiple criteria.

1.1.1 Identify Criteria and Measures

They know that the first thing they must do is identify the **criteria** of a cell phone plan that are important to Isabelle's family. From discussions she has had with her mother and father, Isabelle knows that the criteria that are important to them are the cost and other important factors. Isabelle and Angelo know that they need to find at least one way to **measure** each of the criteria. For the cost criterion, they decide to use the monthly service charge, number of minutes per month and the minimum length of the contract as the measures. For the important factors criterion, they decide to use whether there is free unlimited texting and the quality of service as the measures.

The monthly service charge and the total minutes allowed per month could be any amount within a reasonable range. These are examples of **continuous** measures. Isabelle and Angelo decide that the data they collect for the other three measures can be grouped into a finite number of categories. For example, the measure "Free unlimited text messaging" has only two categories, "yes" and "no." Thus, "Free unlimited text messaging" is an example of a categorical measure.

Isabelle and Angelo similarly define categories for each of the remaining measures. For quality of service, they decide to use ratings from a consumer magazine. They are only considering plans rated good or better. To rate the quality of service, the magazine asked about common cell-phone problems. They used dropped or disconnected calls, static and interference, and voice distortion to rate quality.

Isabelle's parents were also concerned about being locked into paying for a plan for a long period of time. Therefore, Isabelle and Angelo decided to use minimum contract length as one of the measures of cost. Minimum contract length is the shortest time a customer must remain with a particular plan to avoid paying a fee to cancel the service. The plans under consideration had only three different minimum contract lengths, 6 months, 1 year, and 2 years. The categorical variables and their possible values are provided in Table 1.1.1. For each of the categorical measures, Isabelle and Angelo also assigned a number to each category. Those numbers also appear in Table 1.1.1.

Variable	Category	Numeric Value
<i>Free Unlimited Texting</i>	Yes	1
	No	0
<i>Quality of Service</i>	Excellent	2
	Very Good	1
	Good	0
<i>Minimum Contract</i>	6 months	2
	1 year	1
	2 years	0

Table 1.1.1: Categorical variables with categories and numeric values

1.1.2 Collect Data

Isabelle's parents are considering three telephone plans, Trot, ust&t, and Horizon. Isabelle and Angelo collect the data they will need to make their decision. Their data appears in Table 1.1.2.

Plan	Monthly Charge	Minutes per Month	Minimum Contract Length	Free Unlimited Texting	Quality of Service
Trot	\$35	400	6 months	no	Good
ust&t	\$50	500	2 years	yes	Excellent
Horizon	\$60	600	1 year	yes	Very Good

Table 1.1.2: Isabelle and Angelo's cell phone data

1.1.3 Range of Each Measure

Next, Isabelle and Angelo recall that in their mathematics class they learned that it was important to specify the range for each measure. That range will be used to scale the values within the range. There are two continuous measures, monthly service charge and number of minutes allowed per month. For each measure, they decide to use the range of the data they collected. For the categorical variables, they decide to define the range as the specific possible values, because there are only two or three of them.

1.1.4 Rescale Measures to a Common Unit

For a number of reasons, it would be difficult to compare the three plans using this raw data. The first reason is that comparing a \$10 difference in the monthly service charge to a minimum contract length of 2 years or 1 year is like comparing apples to oranges. In order to avoid that problem, operations researchers scale the range of each measure creating a common unit that varies from 0 to 1. Zero always represents the worst value and 1 the best value.

For both of the continuous measures, Isabelle and Angelo decide to use a proportional scale. For example, the range they have decided to use for the monthly service charge measure is from \$35 to \$60. This is a range of \$25. The smallest possible value here is the best option, so \$35 is converted to a common unit value of 1. Similarly, the largest possible value of the monthly service charge is the worst option, so \$60 is converted to 0.

Now, what should be done with the \$50 monthly service charge?

Isabelle and Angelo must assign an appropriate common unit score to the \$50 monthly service charge. They realize that they need to decide where \$50 lies when it is compared to the best and worst options for monthly service charge.

- Q1. Is \$50 closer to the best or the worst option?
- Q2. How far from the best option is \$50? How far from the worst?
- Q3. How far apart are the best and worst options?

Using the answers to these three questions, Isabelle and Angelo solve a proportion to arrive at the common unit value for the monthly service charge of \$50.

$$\frac{10}{25} = \frac{x}{1}$$

$$x = 0.4$$

If Isabelle and Angelo were considering a plan with a \$45 monthly service charge, what is the common unit value for \$45?

Isabelle and Angelo computed the common unit values for the number of minutes allowed each month in the same way.

For the three categorical measures, Isabelle and Angelo again assigned a common unit value of 0 to the worst and 1 to the best. There were only two possibilities for the free unlimited texting measure, so there was nothing else to do. However, for the other two categorical measures, there was something between the best and worst values. For each of these measures, the value in the middle was assigned a common unit value of 0.5.

- Q4. Verify that 0.5 is the appropriate common unit value for a quality measure of very good and a minimum contract length of 1 year if the common unit is assigned proportionally in each case.

Table 1.1.3 contains all of the common unit values for each of the five measures for each of the three plans.

PLAN	Monthly service charge	Minutes per month	Free unlimited texting	Quality of service	Minimum length of contract
Trot	1	0	0	0	1
ust&t	0.4	0.5	1	1	0
Horizon	0	1	1	0.5	0.5

Table 1.1.3: Cell phone data converted to a common unit

When Isabelle and Angelo looked at these results, they noticed that each plan received the top common unit value of 1 on two of the measures. They also noticed that each plan received at least one bottom common unit value of 0. Since the Trot plan got 3 zeros, they thought they could eliminate that plan. However, they weren't sure how to choose between the other two plans.

Angelo thought about using the total of all of the common units to get a total score for each plan. Using his system, Angelo got the following scores:

Trot: 2
ust&t: 2.9
Horizon: 3

Anna thought it would be more meaningful to compute the average common unit score for each plan. When she did so, she obtained the following averages:

Trot: 0.40
ust&t: 0.58
Horizon: 0.60

- Q5. Would it make any difference whether Anna and Angelo used the sum or the average? Explain.

In either case, on the basis of their work, Horizon would be slightly preferred over ust&t, with Trot a distant third. But then they worried about whether some measure was more important than another. For example, Anna remembered that her parents were really worried about the monthly service charge, and not as worried about the length of the contract. They decide that they need a system that does not treat all of the measures equally, as the sum and average do. They need a system that weights the measure according to how important they are to Anna's parents.

1.1.5 Conduct an Interview to Calculate Weights

In order to learn how important each measure is to Anna's parents, Anna and Angelo decide to interview them. First, they need to know which measure is the most important to them. To find out, they ask Anna's parents to rank the five measures in their order of importance. Table 1.1.4 shows their rank ordering of the measures.

MEASURE	Monthly charge	Minutes	Free texting	Quality	Contract length
RANK	1	4	2	3	5

Table 1.1.4: Rank-order of the measures according to Anna's parents

Then Anna and Angelo needed to assign weights to each measure that capture more than the order of importance. They also need a sense of how important the measures are with respect to one another. For example, if one measure is twice as another, then the assigned weights should reflect the strength of that difference. In their math class, they learned a technique of assigning points that can be used to determine the proper weights. The technique continues the interview.

Isabelle and Angelo ask Mr. and Mrs. Nueva to assign 100 points to the measure they ranked number 1. Then they asked them to assign a number of points less than 100 to the second-ranked measure, free unlimited texting. In doing so, they asked Isabelle's parents to pick a number that reflected how important it was compared to the number one ranked measure. Mr. and Mrs. Nueva chose to assign 80 points to free unlimited texting. The interview continued until a number of points had been assigned to each of the five measures. Table 1.1.5 shows the points assigned to each measure.

MEASURE	Monthly charge	Minutes	Free texting	Quality	Contract length
RANK	1	4	2	3	5
POINTS	100	60	80	70	40

Table 1.1.5: Points assigned to each of the measures

Now, Isabelle and Angelo totaled all of the assigned points and divided the point assignment for each measure by the total. This number is the weight of that measure. Table 1.1.6 shows how the weight of the monthly service charge was calculated, as well as the calculated weight for each of the other measures.

MEASURE	Monthly charge	Minutes	Free texting	Quality	Contract length	Total
RANK	1	4	2	3	5	
POINTS	100	60	80	70	40	350
WEIGHT	$100/350=0.29$	0.17	0.23	0.20	0.11	1.00

Table 1.1.6: A weight is calculated for each measure.

Q6. Verify that all of the weights in Table 1.1.6 are correct.

- Q7. What is the largest weight? Which is the smallest?
 Q8. What is the range of the weights, from largest to smallest?
 Q9. What is the ratio of the largest weight to the smallest weight?
 Q10. What should this ratio mean in the context of the decision?

1.1.6 Calculate Total Scores

Now, a total score for each plan can be calculated. The total score is an example of a **weighted average**. Each common unit value from Table 1.1.3 is multiplied by the corresponding weight from Table 1.1.6. Then for each plan, those products are added together to get the total score. Table 1.1.7 shows the results of these computations. Notice that this weighted average captures how important the various measures are to Isabelle's parents. Notice also that on the basis of this weighted average approach, *ust&t* is preferred over Horizon by Isabelle's parents.

PLAN	Monthly service charge (0.29)	Minutes per month (0.17)	Free unlimited texting (0.23)	Quality of service (0.20)	Minimum contract (0.11)	Total Score
Trot	$1(.29)=0.29$	$0(.17)=0$	0	0	0.11	0.40
<i>ust&t</i>	$0.4(.29)=0.12$	$0.5(.17)=0.09$	0.23	0.20	0	0.64
Horizon	0	0.17	0.23	$0.5(.20)=0.10$	$0.5(.11)=0.06$	0.56

Table 1.1.7: A weighted total score is computed for each plan.

- Q11. Would everyone's score results lead to the same preferred choice? Explain.

1.1.7 Determine Strengths/Weaknesses and Make Final Decision

Isabelle and Angelo decide to examine their results, because the total scores of *ust&t* and Horizon were so close. They are also concerned, because their weighting system produced a different result.

- Q12. For which measures did *ust&t* have a higher weighted score than Horizon? For which did Horizon outscore *ust&t*?
 Q13. What were the ranks of the measures where *ust&t* scored higher than Horizon? What were the ranks for the measures where Horizon was higher?

When Isabelle and Angelo compare *ust&t* with Horizon, they see that *ust&t* had higher weighted scores for the first and third ranked measures. Horizon scored higher on the fourth and fifth ranked measures. *ust&t* and Horizon were tied on the second ranked measure. *ust&t* scored better on two of the three most important measures. In contrast, Horizon scored better only on the two least important measures. Therefore, Isabelle and Angelo believe that their weighting system did what it was supposed to do. They decide to recommend the *ust&t* plan to Isabelle's parents.