

**Objective**

In this lesson,  
you will:

- Solve linear systems by using substitution.



**SCENARIO** The Outdoor Club at school is going on a hiking trip and is making trail mix as part of the food that they will take. The trail mix will be made up of nuts and dried fruits, such as raisins, dried cherries, and banana chips. The nuts cost \$4.50 per pound and the dried fruits cost \$3.25 per pound. The group can spend \$15 on the trail mix.

**Key Terms**

- standard form of a linear equation
- substitution method

**Take Note**

A linear equation is in **standard form** if it is written as  $Ax + By = C$ , where  $A$ ,  $B$ , and  $C$  are constants and  $A$  and  $B$  are not both zero.

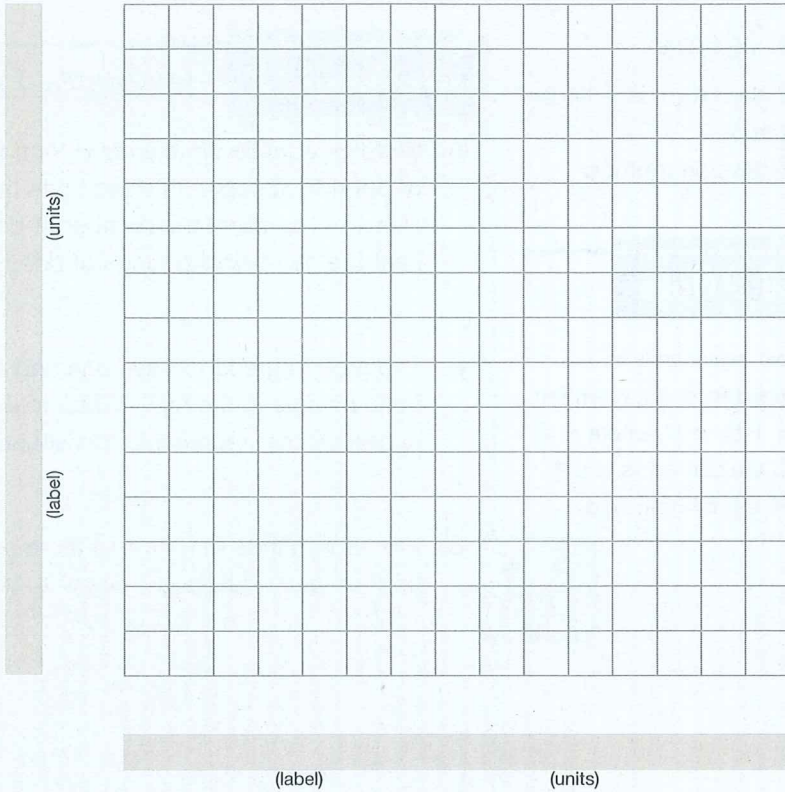
**Problem 1 Making Trail Mix**

- Write an equation in standard form that relates the numbers of pounds of nuts and dried fruits that can be bought for \$15. Use  $x$  to represent the number of pounds of nuts and  $y$  to represent the number of pounds of dried fruits that can be bought.
- The group agreed to have one and a half times as much dried fruits as nuts in the mix. Write an equation in  $x$  and  $y$  as defined in part (A) that represents this situation.
- Will two pounds of nuts and three pounds of dried fruits satisfy both of your equations? Show all your work.
- Will two and one quarter pounds of nuts and one and a half pounds of dried fruits satisfy both of your equations? Show all your work.

## Problem 1 Making Trail Mix

- E. Create a graph of both equations on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



- F. Can you determine the solution of this linear system exactly from your graph? Use a complete sentence to explain your answer.

## Investigate Problem 1

1. Estimate the point of intersection from your graph.

Check your point in each of the equations. Is your point the solution of the linear system?

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2. **Just the Math: Substitution Method** In many systems it is difficult to determine the solution from the graph, so there is an algebraic method for finding the solution. Consider the linear system for this problem situation:

$$4.50x + 3.25y = 15$$

$$y = 1.5x.$$

Because  $y$  is equal to  $1.5x$ , we can substitute  $1.5x$  for  $y$  in the first equation.

$$4.50x + 3.25y = 15$$

$$4.50x + 3.25(1.5x) = 15$$

You now have an equation in  $x$  only. Solve this equation for  $x$ . Show all your work.

### Take Note

It does not matter which equation from the linear system that you use to find the value of  $y$ . You could have used the equation  $4.50x + 3.25y = 15$  to find the value of  $y$ .

Now that you have the  $x$ -value of the solution, find the  $y$ -value by substituting your result for  $x$  into the equation  $y = 1.5x$ . Show all your work.

So, the solution to the linear system is  $(1.6, 2.4)$ . Is this solution confirmed by your graph?

3. Interpret the solution of the linear system in the problem situation. Use a complete sentence in your answer.





## Investigate Problem 1

4. How many pounds of trail mix will the club have? Use a complete sentence to explain how you found your answer.



## Problem 2 Hiking the Trail



The Outdoor Club splits up into two smaller groups to hike the trail. The first group leaves the beginning of the trail and hikes at a rate of 2.5 miles per hour. The second group leaves 30 minutes later and hikes at a rate of 2.5 miles per hour.

- A.** Write an equation for the first group that gives the distance hiked  $y$  in miles in terms of the amount of time  $x$  in hours that the group has been hiking.
- B.** How far will the first group have traveled after 30 minutes of hiking? Show your work and use a complete sentence in your answer.
- C.** Write an equation for the second group that gives the distance hiked  $y$  in miles in terms of the amount of time since the first group started hiking  $x$ .
- D.** How far will each group have traveled 45 minutes after the first group started hiking? Show all your work and use a complete sentence in your answer.

How far will each group have traveled after 2 hours? Show all your work and use a complete sentence in your answer.

- E.** Will the second group catch up to the first group? Use complete sentences to explain your reasoning.

## Investigate Problem 2

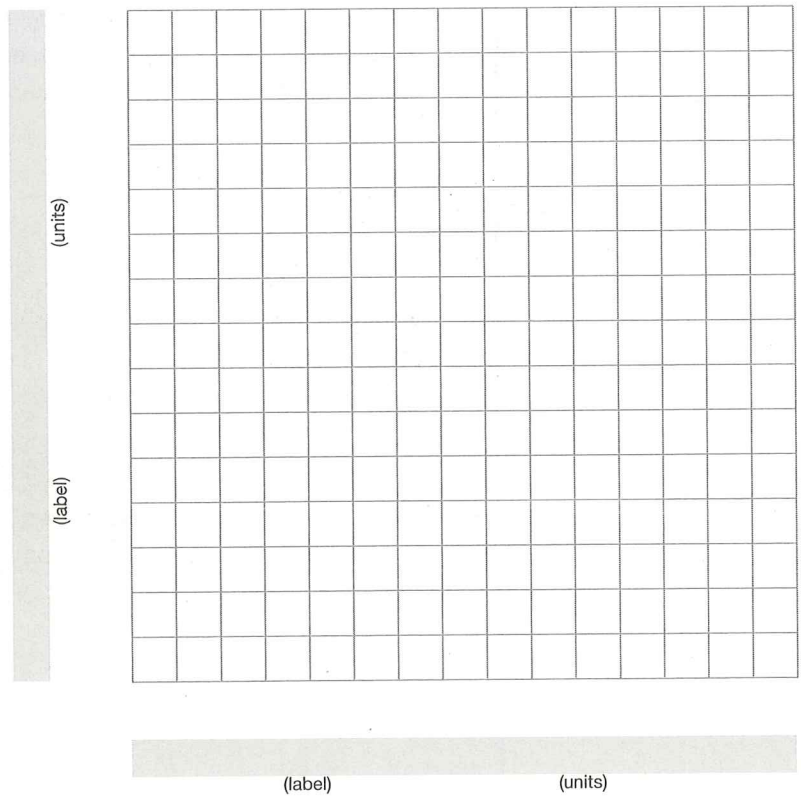
1. Solve the linear system by using the substitution method. First, write your system below.

Next, because you have an expression for  $y$  in terms of  $x$ , substitute your expression for  $y$  from the first equation into the second equation.

Now solve the equation for  $x$ . What is the result? Use a complete sentence in your answer.

2. Create a graph of your linear system on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



## Investigate Problem 2

3. What is the relationship between the lines in the graph? Use a complete sentence in your answer.
4. What is the solution of the linear system? Use a complete sentence in your answer.
5. What is the result when you try to algebraically solve a linear system that has no solution? Use a complete sentence in your answer.



## Problem 3 Camping



Another community group joins the Outdoor Club at the campsite. The new group has rented six tents and twenty four sleeping bags for \$186. The Outdoor Club rented from the same place and rented eight tents and thirty sleeping bags for \$236. Each tent costs the same, and each sleeping bag costs the same.

- A. For each group, write an equation in standard form for this problem situation. Use  $x$  to represent the cost of one tent in dollars and use  $y$  to represent the cost of one sleeping bag in dollars.
- B. Without solving the linear system, interpret the solution of the linear system in part (A). Use a complete sentence in your answer.
- C. Can you tell from looking at the equations whether the linear system has a solution? Use a complete sentence to explain your reasoning.



## Investigate Problem 3

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1. How does this linear system differ from the linear systems that you wrote in Problems 1 and 2? Use complete sentences in your answer.
2. To solve this linear system by using the substitution method, what do you think you would have to do first? Use a complete sentence in your answer.
3. Write the equation for the community group in slope-intercept form. Show all your work.
4. Now, use the substitution method to solve the linear system. Begin by substituting your expression from Question 3 for  $y$  in terms of  $x$  into the equation for the Outdoor Club.

$$8x + 30(\text{ }) = 236$$

Now solve this equation for  $x$ . Show all your work.

Finally, find the value for  $y$ . Show all your work.

5. Check your answer algebraically. Show all your work.
6. Interpret the solution of the linear system in the problem situation. Use complete sentences in your answer.

### Take Note

Whenever a product involves a sum, such as  $4(x + 3)$ , you must use the distributive property to simplify:

$$4(x + 3) = 4(x) + 4(3).$$

## Investigate Problem 3



7. If possible, solve each linear system by using the substitution method. Show all your work and use a complete sentence in your answer. Then check your answer algebraically.

$$4x + 3y = 10$$
$$y = 2x$$

$$y = 2x - 1$$
$$y = 3x + 1$$

$$8x - 2y = 7$$
$$2x + y = 4$$

$$6x + 3y = 5$$
$$y = -2x + 1$$

