

Objectives

In this lesson, you will:

- Determine the number of solutions of a linear system.
- Identify parallel and perpendicular lines.



SCENARIO A process engineer is performing a time study on a construction site. As part of the study, the work rates of a novice (beginner) bricklayer and a more experienced bricklayer are being recorded. At the beginning of the study, the novice had put 1510 bricks into place and was setting the bricks in place at a rate of thirty eight bricks per hour. The experienced worker started the job after the novice and had put 960 bricks into place so far and was setting the bricks in place at a rate of sixty bricks per hour.

Key Terms

- system of linear equations
- linear system
- solution
- point of intersection
- parallel lines
- perpendicular lines
- reciprocals

**Problem 1 The Novice and the Pro**

- A. For each worker, write an equation that gives the total number of bricks y set in place in terms of the time x in hours after the beginning of the time study.
- B. After eight hours of the time study, how many bricks in all will each worker have set into place? Show all your work and use complete sentences in your answer.

Which worker has set more bricks into place after eight hours of the time study? Use a complete sentence in your answer.

- C. After forty hours of the time study, how many bricks in all will each worker have set into place? Show all your work and use complete sentences in your answer.

Which worker has set more bricks into place after forty hours of the time study? Use a complete sentence in your answer.

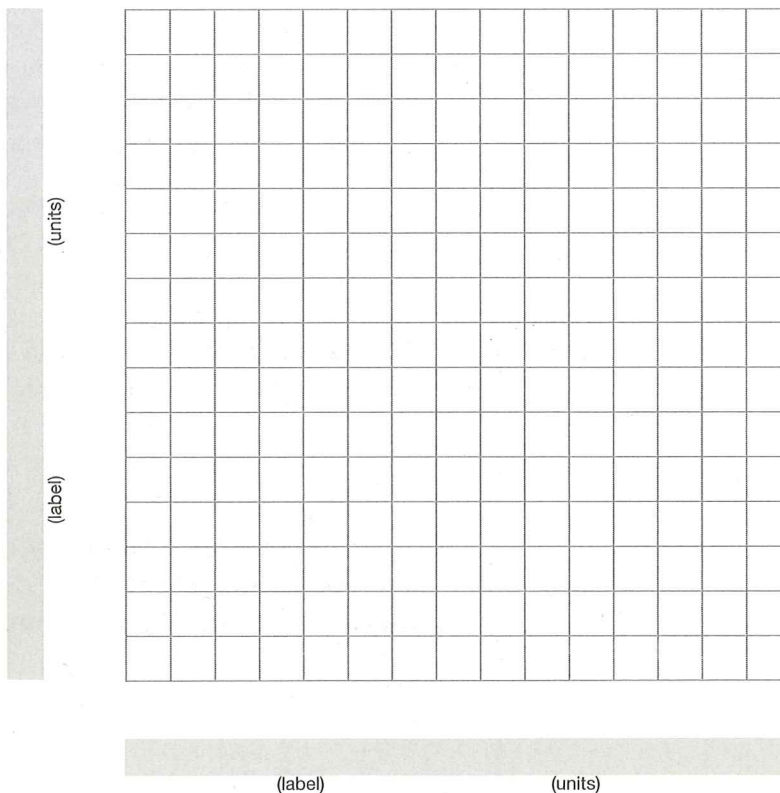
Problem 1 The Novice and the Pro

- D. Find the number of hours that the time study would need to run in order for each worker to set a total of 2460 bricks. Show all your work and use complete sentences in your answer.

Investigate Problem 1

1. Create a graph of both equations on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



2. Find the amount of time that it will take in the time study for the number of bricks set by each worker to be the same. Use a complete sentence to explain how you found your answer.

Investigate Problem 1

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3. What does the slope of each line represent in this problem situation? Use a complete sentence in your answer.

Which worker sets bricks faster? How do you know?
Use a complete sentence in your answer.

4. What does the y -intercept of each line represent in this problem situation? Use a complete sentence in your answer.

How do the y -intercepts of the lines compare? What does this mean in the problem situation? Use complete sentences in your answer.

5. Just the Math: Systems of Linear Equations

In this lesson and in Lesson 7.1, you considered the graphs of two linear equations together. When you do this, you form a **system of linear equations** or a **linear system**. Write the linear system represented by the graph in Problem 1.

6. Just the Math: Solution of a Linear System

The **solution** of a linear system is an ordered pair (x, y) that is a solution to *both* equations in the system. Graphically, the solution is the **point of intersection** of the system. What is the solution of the linear system in this problem situation? Use your graph to help you. Write your answer using a complete sentence.

Algebraically, verify that the ordered pair is a solution of your system. Remember that the ordered pair needs to be a solution of both equations.

Take Note

Recall that to algebraically verify that an ordered pair is a solution of an equation, substitute the values given by the ordered pair for x and y in the equation.

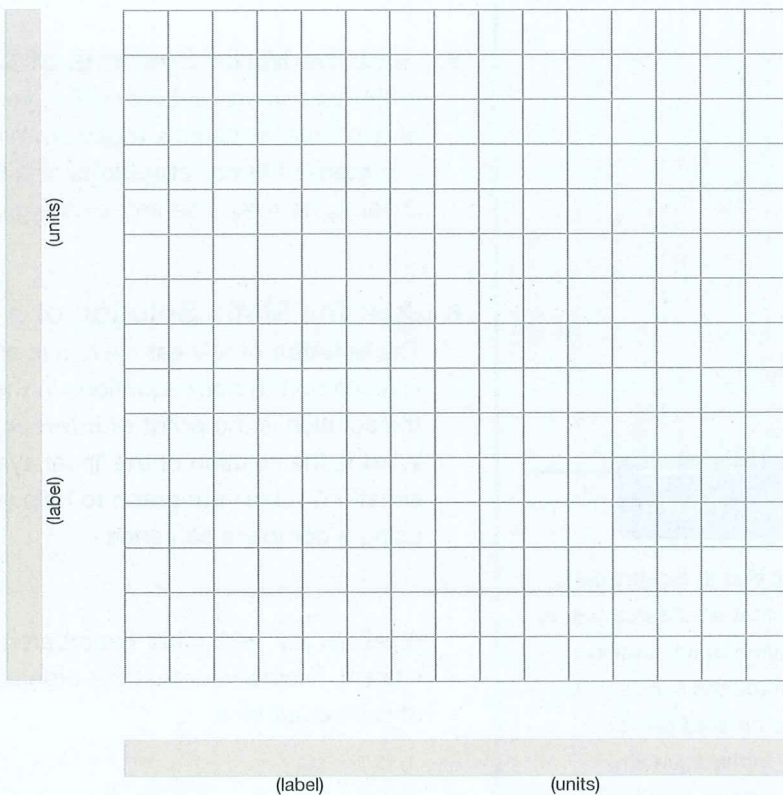
These values should give you a true statement.

Problem 2 The Pros



- A. Another experienced bricklayer is having her time recorded as a part of the time study. At the beginning of the study, this worker had set 600 bricks so far and can set 60 bricks in one hour. Write an equation that gives the total number of bricks y set in place in terms of the time x in hours after the beginning of the time study.
- B. Write a linear system that shows the total number of bricks set in terms of time for both experienced workers.
- C. Create a graph of the linear system on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

Variable quantity	Lower bound	Upper bound	Interval



Investigate Problem 2

7

1. What does the slope of each line represent in this problem situation? Use a complete sentence in your answer.

Which worker sets bricks faster? How do you know?
Use a complete sentence in your answer.

2. What does the y -intercept of each line represent in this problem situation? Use a complete sentence in your answer.

How do the y -intercepts of the lines compare? What does this mean in the problem situation? Use complete sentences in your answer.

3. Does there appear to be any point of intersection of the lines?

4. Use complete sentences to describe how the lines are related to each other.

5. **Just the Math: Parallel Lines** The lines that you graphed in part (C) are *parallel lines*. Two lines in the same plane are **parallel** to each other if they do not intersect. What can you conclude about the slopes of parallel lines? Use a complete sentence in your answer.

6. Does the linear system for the two experienced workers have a solution? Use complete sentences to explain your reasoning.

7. Will the two experienced workers ever set the same number of bricks during the time study? Use complete sentences to explain your reasoning.

Investigate Problem 2

8. **Just the Math: Number of Solutions of a Linear System** So far in this lesson, we have seen a linear system with one solution and a linear system with no solution. Use complete sentences to describe the graphs of these kinds of linear systems.

Consider the following linear system:

$$y = 2x - 4 \text{ and } y = -2(2 - x).$$

Complete the table of values for this linear system.

Expression

x	$2x - 4$	$-2(2 - x)$
-5		
0		
5		
10		
12		
15		

What can you conclude about the number of solutions of this linear system? Use a complete sentence in your answer.

Because every point on the graph of $y = 2x - 4$ is on the graph of $y = -2(2 - x)$, we can say that this system has an *infinite number* of solutions. Use a complete sentence to explain why you think this is true.



Problem 3 When Is the Job Done?



- A. The experienced bricklayer who sets bricks at a rate of 60 bricks per hour and has set 960 bricks so far must set approximately 20,000 additional bricks before the job is done. Write an equation that gives the total number of bricks y left to set in terms of the time x in hours after the beginning of the time study.

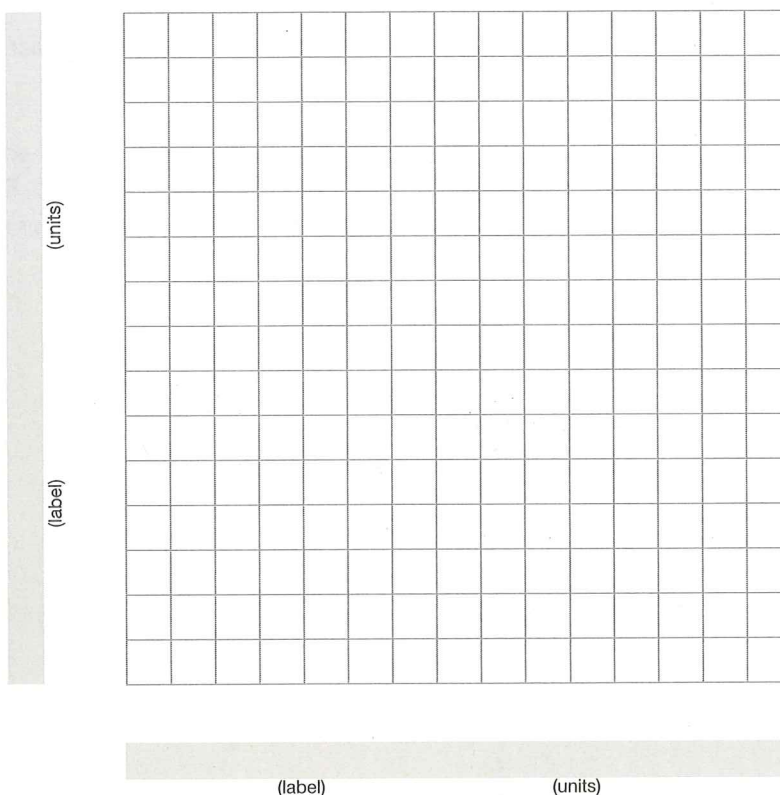
Problem 3 When Is the Job Done?

- B. Form a linear system with the equation in part (A) and the equation from Problem 1, part (A) that gives the total number of bricks set by this worker in terms of the time after the beginning of the time study.

Investigate Problem 3

1. Create a graph of the linear system on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly

Variable quantity	Lower bound	Upper bound	Interval



2. What does the point of intersection of the lines represent?
3. Compare the slopes of the lines. Use a complete sentence in your answer.

Investigate Problem 3

4. Are the lines *perpendicular*? That is, do they intersect at a right angle?
5. Consider the graph of your linear system and the equations of the lines. What do you notice about the slopes of perpendicular lines?

Take Note

Two numbers are **reciprocals** if their product is 1.

6. **Just the Math: Perpendicular Lines** A property of **perpendicular lines** is that the product of their slopes must be -1 . So, this means that the slopes must have opposite signs and must be **reciprocals** of each other. For instance, the lines

$$y = -3x + 4 \text{ and } y = \frac{1}{3}x + 1 \text{ are perpendicular because}$$

$-3\left(\frac{1}{3}\right) = -1$. Algebraically show that the lines in your graph in Question 1 are *not* perpendicular. Show your work.

7. Determine whether the graphs of each pair of equations are parallel, perpendicular, or neither. Show your work and use a complete sentence to explain your reasoning.

$$y = \frac{2}{3}x + 4 \text{ and } y = -\frac{3}{2}x + 1$$

$$y = 5x - 4 \text{ and } y = -5x + 4$$

$$y = 4x \text{ and } y = \frac{1}{4}x - 2$$

$$y = -1.8x + 15 \text{ and } y = 6 - 1.8x$$

